Received: Aug ‘22
Revised: Aug ‘22
Accepted: Sep ‘22

A STUDY ON THE FIRST EIGENVALUE OF THE *P*-LAPLACIAN AND CRITICAL SETS OF HARMONIC FUNCTIONS BY DEFINING GEOMETRIC *P*-LAPLACIAN ON RIEMANNIAN MANIFOLDS

Manisha Rajput [1] and Dr. Dharmendra Saxena[2]
[1]Research Scholar Mansarovar Global University, Sehore (M.P.)
[1]Faulty of Govt. College Timarni
[2]Professor of Mathematics, Mansarovar Global University, Sehore (M.P.)

ABSTRACT

The p-Laplacian, sometimes known as the p-Laplace chairman, is a second-request quasilinear elliptic deficient differential executive in mathematics. When p is permitted to surpass 1p, the Laplace head hypothesis is nonlinear. This is a sizeable chunk of the range of the weighted p-Laplacian on a complete Riemannian complex with changing mathematics, which propels as an invariant sum under any mathematical stream. The basic Eigen value of the p-Laplacian on an n-layered closed Riemannian complex, which is estimated by a summarised mathematical stream, takes into account variety plans, monotonicity, and differentiability. Mathematicians and physicists have been particularly interested in the range of the Laplacian on noncom pact non-complete manifolds over the past thirty years due to the applications in flexibility and various fields of the study of the phantom features of the Dirichlet Laplacian in infinitely expanded regions. The basic points for p-symphonious guides between Riemannian manifolds are investigated using p-Laplacian PDEs in differential mathematics, and the width of the manifolds is determined using p-Laplacian on Riemannian manifolds eigenvalue problems. The study of the bizarre characteristics of the direct Laplacian on an area in a Euclidean space or a complex has made substantial use of the concept of self-adjoin administrators.

Keywords: Eigen Value, *P*-Laplacian, Harmonic Functions, Geometric, Riemannian, Manifolds, etc.

1. INTRODUCTION

The p-Laplacian is a second-request quasilinear elliptic incomplete differential administrator that is sometimes sometimes referred to as the p-Laplace administrator. The Laplace administrator's nonlinear hypothesis, where p might be any number more pronounced than $1<p<\infty$. A symphonious capacity is a two times constantly differentiable capacity $f: U \rightarrow R$, where U is an open subset of R^n that satisfies Laplace’s condition in science, mathematical physical science and the speculation of stochastic cycles. It is a huge part of the range of the weighted p - Laplacian on a total Riemannian complex with changing math that the range progresses as an invariant sum as the space does under any mathematical stream. All through, we will accept a n-layered full Riemannian complex (M, g, $d\mu$) with a weighted measure $d\mu = e^{-\phi}\, dv$ and a potential capacity $\phi \in C\infty(M, d\mu)$, whose metric $g = g\ (t)$ progresses along either the Ricci-symphonious stream or the volume-saving Ricci-consonant stream.

2. THE *p*-LAPLACIAN'S FIRST EIGENVALUE ON EVOLVING GEOMETRY AND APPLICATIONS

The variety method, monotonicity, and differentiability are looked at for the primary Eigen value of the p-Laplacian on an n-layered closed Riemannian complex, which is estimated by a mathematical stream that has been condensed. It is demonstrated that, under certain mathematical hypotheses, the first nonzero Eigen value is differentiable everywhere and monotonically non-lessening along the stream. These findings offer a cohesive method for handling the examination of Eigen value fluctuations and applications in various mathematical fields.

2.1 Geometric flow

(M, g) is a n-layered shut Riemannian complex ($n > 1$). Let $g\ (x,\ t)$ be a one-boundary assortment of estimations for t [0, T] and $x \in M$. We say that g (x, t) is a summarized mathematical stream assuming it advances as indicated by the situation g (x,0)= $g_0(x)$ where $0<T<T\varepsilon$ is the maximum season of presence is T, where h is a general time-subordinate symmetric 2-tensor, and when the stream initially explodes. H is acknowledged to be smooth in both t and x for this circumstance. Since g is smooth in the two variables, this is obvious. In (1), the scaling factor 2 is not necessary; but, in some situations, the negative sign may be necessary to maintain the

stream moving forward or backward according to schedule.

$$\frac{\vartheta}{\vartheta} g(x,t) = -2h(x,t), \quad (x,t) \epsilon M \times [0,T] \quad \textbf{(1)}$$

The Ricci stream, with h being the Ricci shape tensor, and the mean twist stream, with $h = H$, are two normal instances of mathematical streams in this class (where H is the mean curve and is the second critical construction on M). Yamane stream, Ricci-consonant stream, and RicciBourguignon stream are among models. On tensor h, a roundedness necessity can be forced. Frankly, such roundedness and sign presumptions on h are held as long as the stream remains, so the estimations are steady all of the time. In particular, if $-K1g \leq h \leq K2g$, where $g(t)$, $t \in [0, T]$ addresses the stream, then

$$e^{-K_i T} g(0) \leq g(t) e^{K_2 T} g(0)$$

Consider the advancement of a vector structure $|X|g = g(X, X)$, $X \in TxM$. to see the last limits. We get $|\partial tg(X, X)| \leq K2g(X, \boldsymbol{X})$, from (1) and the roundedness of the tensor *h,* which involves (by planning from $t1$ to $t2$).

$$\left| \log \frac{g(t_2)(X,X)}{g(t_1)(X,X)} \right| \leq K_2 t \Big|_{t_1}^{t_2}$$

Taking the outstanding of this check with $t_1 = 0$ and $t_2 = T$ gives $|g(t)| \leq e^{K_2 T} g(0)$, which demonstrates that the estimation is consistently restricted. Thuly, accepting the roundedness supposition that is valid,

$$-K_1 g \leq h \leq K_2$$

2.2 Eigen value of *p* –Laplacian

$$\Delta p, gf(x) := \operatorname{div}(|\nabla f|^{p-2} \nabla f)(x)$$

For $p \in [1, \infty)$, where div is the distinction administrator, and the adjoint of tendency (graduate) for the L^2 - standard prompted by g on the space of differential constructions at $p = 2$, $\Delta_{2,g}$ is the standard Laplace-Beltrami administrator. The nonlinear Eigen esteem issue is settled by the Eigen esteems and relating Eigen elements of $\Delta p, g$ fulfill the accompanying nonlinear eigen value problem

$$\Delta_{p,g} f = -\lambda |f|^{p-2} f, f \neq 0. \quad (2)$$

To stay away from the differentiability supposition on the main Eigen esteem and the going with Eigen work for the circumstance $p \neq 2$, we will utilize strategies illustrated in under the Ricci stream to research the variety and monotonicity of $\lambda_{p,1}(t) = \lambda_{p,1}(t, f(t))$, where $\lambda_{p,1}(t, f(t))$, and $f(t)$ are smooth. The underlying Eigen worth's development and monotonicity recipes construed here are free of the Eigen capacity's advancement. A couple of standardization conditions should be met for the Eigen capacity to work. On advancing manifolds with or without rhythmic movement suspicions, there are an assortment of results for the advancement and monotonicity of Eigen upsides of the Laplace administrator. With entropy methods, one can find under the Ricci stream, under the Ricci-symphonious stream, and along a remarkable mathematical stream. The investigation of the properties of p - Laplacian Eigen esteems on creating buildings is as yet in its beginning phases. The significant objective of this work is to check whether the known elements of λp, 1 on static estimation and $p = 2$ on developing measurement can be stretched out to various mathematical streams. Notwithstanding, we need to give a consolidated equation that might be utilized for this reason on time-subordinate perceptions. There are various interesting outcomes connecting with λp, 1 conduct.

3. IN DIMENSION TWO, GEOMETRIC PROPERTIES OF A SOLUTION TO THE ANISOTROPIC *p*-LAPLACE EQUATION

We investigate answers for condition (3) with a constantly elliptic and Lipchitz consistent symmetric cross section. We think about $u \in W^{1,p}loc\ (\Omega)$ answers for the accompanying savage elliptic condition, which we will call the anisotropic p-Laplace.

$$div\ (|A \nabla v . \nabla v|^{(p-2)/2} A \nabla \mu) = 0 \ in \ \Omega \quad (3)$$

Where is a two-layered area, p approaches $1<p<\infty$, and $A = A(x)$ is a symmetric organization that fulfills the uniform flexibility and Lipchitz progression speculations. The Euler condition for the variety basic is Equation (3).

$$J(u) = \int_{\Omega} |A \nabla u \cdot \nabla u|^{p/2} dx$$

Its significance originates from an assortment of uses including composite materials, for example, nonlinear dielectric composites, whose nonlinear it is shown by the supposed power-law. Numerous things are considered in the present circumstance, including the nearby conduct of arrangements and the development of level lines and basic focuses. To begin with, the Hartman and Winner hypothesis [HW] shows that $u(x)$-$u(x^0)$ is asymptotic to a homogeneous symphonious polynomial of $x^0 \in \Omega$, up to a straight difference in arranges that delivers

A (x^0)= const. I, and that this asymptotic reaches out over to initially demand subsidiaries.

From this crucial truth, one can derive that in the event that u is non-indistinguishably consistent; its basic focuses are isolated. Besides, in the event that x^0 is a zero of variety m for ∇u, the level set {x | u(x) = u (x^0)} is made of precisely m +1 basic bends converging at x^0 just at x^0. The number and multiplicities of basic marks of an answer can then be determined utilizing the properties of its Dirichlet information [A1], [A2], or different kinds of limit information [AM1]. At the point when the coefficient network and is just confined quantifiable [AM2], such outcomes have been extended to frail arrangements u to (6).

4. ESTIMATES FOR THE HEAT KERNEL EIGENVALUE AND INEQUALITIES FOR THE P-LAPLACIANS ON RIEMANNIAN MANIFOLDS P-LAPLACIANS ON RIEMANNIAN MANIFOLDS: INEQUALITIES

In order to examine the spectral characteristics of the linear Laplacian on a domain in a Euclidean space or a manifold, the theory of self-adjoint operators has been employed extensively. Mathematicians often have an interest in the Laplacian spectrum on compact manifolds (with or without boundary) or non-compact complete manifolds since in both of these cases the linear Laplacians can be extended out to self-adjoint operators. The study of the horrifying characteristics of the Dirichlet Laplacian in infinitely broad regions has applications in a variety of fields, including versatility, acoustics, electromagnetism, quantum physics, and others. As a result, mathematicians and physicists have been particularly interested in the Laplacian's range on noncompact non-complete manifolds over the past 30 years. Recently, the inventor proved that there is a discrete range of the direct Laplacian on a class of 4-layered rotationally symmetric quantum layers, which are non-minimal non-complete manifolds, with the right mathematical assumptions.

5. CONCLUSION

The p-Laplacian, also known as the p-Laplace operator, is referred to as a second-order quasilinear elliptic partial differential operator by mathematicians. It is a nonlinear generalisation of the Laplace operator with p ranging from 1 to p. For the first eigenvalue of the p-Laplacian on an n-dimensional closed Riemannian manifold, which is measured by a generalised geometric flow, the variation formulae, monotonicity, and differentiability are examined. The spectral characteristics of the linear Laplacian on a manifold or a domain in Euclidean space have been thoroughly investigated with the aid of the theory of self-adjoint operators. Mathematicians typically have an interest in the Laplacian spectrum on compact manifolds (with or without boundary) or non-compact complete manifolds in order to extend the linear Laplacians to self-adjoint operators. The p-Laplace operator is extensively studied since it depicts many of the actions that occur in nonlinear analysis. Singular and nonsingular boundary value problems, which are present in many disciplines of mathematical physics, are one of their applications.

REFERENCES

1. J. Wu (2011). First eigenvalue monotonicity for the p-Laplace operator under the Ricci flow, Acta Math. Sinica 27, pp. 1591-1598.
2. Abolarinwa (2019). Eigenvalues of Weighted-Laplacian under the extended Ricci flow, Adv. Geom. 19, pp. 131-143.
3. L. Damascelli and F. Pacella, Monotonicity and symmetry of solutions of p-Laplace equations, 1 < p < 2, via the moving plane method, Ann. Scuola Norm. Sup. Pisa Cl. Sci. 26 (1998), 689–707.
4. Bj¨orn, J. Bj¨orn and N. Shanmugalingam, The Dirichlet problem for p-harmonic functions on metric spaces, J. Reine Angew. Math. 556 (2003), 173–203.
5. M. Bartuˇsek, M. Cecchi, Z. Doˇsl´a and M. Marini, On oscillatory solutions of quasilinear differential equations, J. Math. Anal. Appl. 320 (2006), 108–120.
6. R. P. Agarwal, S. R. Grace and D. O'Regan, Oscillation theory for second order dynamic equations, Ser. Math. Anal. Appl., vol. 5, Taylor & Francis, 2003.